1. DIFFERENTIAL CALCULUS, QUADRATIC EQUATIONS, TRIGONOMETRIC RATIOS & EQUATIONS TEST

Duration – 1 hr **JEE MAINS PATTERN** Total Marks -120

No. of Questions 30 , Marking Scheme: 4 marks for correct answer, -1 for wrong answer.

1.

Range of function $f(x) = |6\sin^{-1}x - \pi| + |6\cos^{-1}x - \pi|$ is

(A) $[0, 9\pi]$ (B) $[0, 3\pi]$ (C) $[\pi, 3\pi]$ (D) $[\pi, 9\pi]$

2.

If $e^x + e^{f(x)} = e$, then the range of f(x) is

(A) $(-\infty, 1]$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $[1, \infty)$

3.

If f(x) = x + tanx and f(x) is inverse of g(x), then g'(x) is equal to

(A) $\frac{1}{1+(g(x)-x)^2}$ (B) $\frac{1}{1+(g(x)+x)^2}$ (C) $\frac{1}{2-(g(x)-x)^2}$ (D) $\frac{1}{2+(g(x)-x)^2}$

4.

The function $y = \frac{x}{1+|x|} : R \to R$ is

A. One-One, Onto B. One-One, Into, Odd C. Many-One, Onto, Odd D. One-One,Onto, Odd

5.

The sum of all the values of 'm' for which the roots x_1, x_2 of quadratic equation $x^2 - 2mx + m = 0$ satisfy $x_1^3 + x_2^3 = x_1^2 + x_2^2$ is

(1) 3/4
(2) 1
(3) 4/3
(4) 9/4
(5) 5/4

6.

For x, the solution of $[x + 2] + [x - 8] > 0$ is

([.] is greatest integer function)

(1) $[3, \infty)$
(2) $[4, \infty)$
(3) [1, 3]
(4) (3, 4)
(5) R

7.

If $f(x) + 2f(1 - x) = x^2 + 2 \ \forall \ x \in R$ then $f(x)$ is given by

(1) $\frac{(x-2)^2}{3}$

(2) $x^2 - 2$

(3) $(x - 2)^2$

(4) $\frac{x^2-2}{3}$

(5) $\frac{x^2}{3} - 2$

8.

If $\tan 1^\circ = t$, the value of $\cos 2^\circ + t \sin 2^\circ$ is

(1) t

(2) 1

(3) $\frac{1}{2}$

(4) 0

(5) –1

9.

If $f(x) = \min(|x|^2 - 5|x|, 1)$ then $f(x)$ is non differentiable at λ points, then $\lambda + 13$ equals

(1) 6

(2) 8

(3) 12

(4) 16

10.

The maximum value of $f(x) = 2bx^2 - x^4 - 3b$ is $g(b)$, where $b > 0$. if b varies then the minimum value of $g(b)$ is

(1) $\frac{3}{2}$

(2) $\frac{9}{2}$

(3) $-\frac{9}{4}$

(4) $-\frac{9}{2}$

11.

The range of the function

$f(x) = (1+\sec^{-1}x)(1+\cos^{-1}x)$ is

(1) $(-\infty,\infty)$

(2) $(-\infty,0]\cup[4,\infty)$

(3) $\{1,(1+\pi)^2\}$

(4) $\{0,(1+\pi^2)\}$

12.

$\tan B = \frac{3\sin A\cos A}{1-3\cos^2 A}$ then tan (A + B) equals

(1) $\frac{-1}{2}\tan A$

(2) 2 cot A

(3) $\frac{1}{2}\tan A$

(4) – 3cotA

13.

A function f from integers to integers is defined as $f(x) = \begin{cases} n+3 & , \text{ n is odd} \\ \frac{n}{2} & , \text{ n is even} \end{cases}$. If k is an odd integer and f(f(f(k))) = 27 then the sum of digits of k is

(1) 3

(2) 6

(3) 9

(4) 12

14.

If $f(x)=\begin{cases} x^2+2 & , \ x<0 \\ 3 & , \ x=0 \\ x+2 & , \ x>0 \end{cases}$, then which of the following is **FALSE**.

(1) f(x) has a local maximum at x = 0

(2) f(x) is strictly decreasing on the left of x = 0

(3) f′(x) is strictly increasing on the left of x = 0

(4) f′(x) is strictly increasing on the right of x = 0

15.

If the thrice repeated roots of equation $x^4 + ax^3 + bx^2 + cx - 1 = 0$ is 1, then a + b + 2c is equal to

(1) 0

(2) 1

(3) –1

(4) 2

(5) –2

16.

The minimum value of $f(x) = x^2 + 2x + \frac{24}{x}$, (where x > 0) is

(1) 12

(2) 16

(3) 20

(4) $7(6)^{4/7}$

(5) $5(8)^{4/5}$

17.

The value of $\int_{-2}^{2} \min\{x-[x], -x-[-x]\}\,dx$ is,

(where [.] represent greatest integer function]

(1) $\frac{1}{2}$

(2) 1

(3) $\frac{3}{2}$

(4) 2

(5) 3

18.

The solution of the equation

$2\cos^4 x + \cos x - 2\cos x \sin^2 x - 3\sin^2 x + 1 = 0$ is

(1) $n\pi \pm \frac{\pi}{4}, n \in I$

(2) $n\pi \pm \frac{\pi}{2}, n \in I$

(3) $n\pi \pm \frac{\pi}{6}, n \in I$

(4) $n\pi \pm \frac{\pi}{8}, n \in I$

(5) $n\pi \pm \frac{\pi}{3}, n \in I$

19.

If a variable tangent to the curve $x^2y = c^3$ makes intercepts a,b on x and y axes respectively, then the value of a^2b is

(1) $27c^3$

(2) $\frac{4}{27}c^3$

(3) $\frac{27}{4}c^3$

(4) $\frac{4}{9}c^3$

(5) $\frac{27}{8}c^3$

20.

$\lim\limits_{x \to \pi/2} \frac{2^{2\cos x} - 1}{x(x - \pi/2)}$ is equal to

(1) $\frac{\log 4}{\pi}$

(2) $\frac{-\log 4}{\pi}$

(3) $\frac{\log 2}{\pi}$

(4) $-\frac{\log 2}{\pi}$

(5) $\frac{-2\log 4}{\pi}$

21.

One of the point on the curve $3y = 6x - 5x^3$, normal at which passes through the origin, is

(1) $(-1, 3)$

(2) $\left(\frac{1}{3}, 1\right)$

(3) $\left(2, -\frac{28}{3}\right)$

(4) $\left(1, \frac{1}{3}\right)$

(5) $\left(2, \frac{28}{3}\right)$

22.

Let $f : R \to R$ is a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x), \forall x \in R$. For this function f, The graph of y =f(x) is not symmetrical about

(a) x = 2 (b) x =10

(c) x =8 (d) None of these

23.

The minimum value of $\sec^{-1}\left(\frac{7-5(x^2+3)}{2x^2+4}\right)$ is $\frac{2\lambda\pi}{\mu}$ then $\lambda + \mu$ is.

(1) 2

(2) 4

(3) 6

(4) 8

(5) 0

24.

Let $f(x) = \begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If y = g(x) is image of y = f(x) in y-axis, then absolute value of f(1) g(1) is

(1) 0

(2) 1

(3) 2

(4) 3

(5) 4

25.

Let g(x) be a polynomial of degree one and f(x) be defined by f(x) = $\begin{cases} g(x), & x \le 0 \\ |x|^{\sin x}, & x > 0 \end{cases}$. If f(x) is continuous satisfying f'(1) = f(–1), then g(x) is

(A) (1 + sin 1)x + 1 (B) (1 – sin 1)x + 1 (C) (1 – sin 1)x – 1 (D) (1 + sin 1)x – 1

26.

Let f: R $\to$ R be a continuous onto function satisfying f(x) + f(–x) = 0, $\forall$ x $\in$ R. If f(–3) = 2 and f(5) = 4 in [–5, 5], then the equation f(x) = 0 has

(A) exactly three real roots (B) exactly two real roots

(C) atleast five real roots (D) atleast three real roots

27.

$\lim\limits_{x\to-\infty}\left(\sqrt{x^2-\frac{2x}{3}}+7+x\right)$

(A) $\frac{22}{3}$ (B) $\frac{16}{9}$

(C) $\frac{17}{3}$ (D) $\frac{8}{3}$

28.

Let $f(x)=\{x\}+\left\{x+[e^x]\right\}+\left\{x+[e^{2x}]\right\}+\left\{x+[e^{3x}]\right\}.....\left\{x+[e^{99x}]\right\}$ then $\lim\limits_{x\to\pi}\left[f(x)\right]$ is equal to

(where { } F.P.F [.] G.I.F)

(A) 14 (B) 28

(C) 7 (D) does not exist

29. If f(x) is a quadratic expression such that f(–3) = f(3) = 0 and f(1) = 3 then $\lim\limits_{x\to0}\frac{\sqrt[3]{f(x)}-\frac{3}{2}}{\ell n\cos x}$

(A) $\frac{1}{9}$ (B) $\frac{2}{3}$

(C) $\frac{2}{9}$ (D) None of these

30.

Range of f(x) = [1+ sinx] + $\left[2+\sin\frac{x}{2}\right]$ + $\left[3+\sin\frac{x}{3}\right]$ + ...+ $\left[n+\sin\frac{x}{n}\right]\forall x\in[0,\pi]$, where [.] denotes the greatest integer function, is

(A) $\left\{\frac{n^2+n-2}{2},\frac{n(n+1)}{2}\right\}$ (B) $\left\{\frac{n(n+1)}{2}\right\}$

(C) $\left\{\frac{n^2+n-2}{2},\frac{n(n+1)}{2},\frac{n^2+n+2}{2}\right\}$ (D) $\left\{\frac{n(n+1)}{2},\frac{n^2+n+2}{2}\right\}$